Evaluation of wavelet domain methods for image denoising

P. Kenterlis[1], **D. Salonikidis**[2]

[1]Department of Electronic Computer Systems, Technological Education Institute of Piraeus, P. Ralli & Thivon 250, 12244 Egaleo, Greece

[2]Department of Automation, Alexander Technological Institute of Thessaloniki, 57400 Thessaloniki, Greece

(P.Kenterlis@mprolab.teipir.gr, denis127@yahoo.gr)

ABSTRACT

Image denoising is a common procedure in digital image processing aiming at the removal of noise, which may corrupt an image during its acquisition or transmission, while retaining its quality. This procedure is traditionally performed in the spatial or frequency domain by filtering. Recently, a lot of methods have been reported that perform denoising on the Discrete Wavelet Transform (DWT) domain. The transform coefficients within the subbands of a DWT can be locally modeled as i.i.d (independent identically distributed) random variables with Generalized Gaussian distribution. Some of the denoising algorithms perform thresholding of the wavelet coefficients, which have been affected by additive white Gaussian noise, by retaining only large coefficients and setting the rest to zero. However, their performance is not sufficiently effective as they are not spatially adaptive. Some other methods evaluate the denoised coefficients by an MMSE (Minimum Mean Square Error) estimator, in terms of the noised coefficients and the variances of signal and noise. The signal variance is locally estimated by a ML (Maximum Likelihood) or a MAP (Maximum A Posteriori) estimator in small regions for every subband where variance is assumed practically constant. These methods present effective results but their spatial adaptivity is not well suited near object edges where the variance field is not smoothly varied. The optimality of the selected regions where the estimators apply has been examined in some research works.

This paper evaluates some of the wavelet domain algorithms as far as their subjective or objective quality performance is concerned and examines some improvements.

KEYWORDS

Discrete Wavelet Transform, denoising, quad-tree decomposition.

1. INTRODUCTION

An image is often corrupted by noise during its acquisition or transmission. Image denoising is used to remove the additive noise while retaining as much as possible the important image features. In the recent years there has been a fair amount of research on filtering and wavelet coefficients thresholding, because wavelets provide an appropriate basis for separating noisy signal from the image signal. These wavelet-based methods mainly rely on thresholding the Discrete Wavelet Transform (DWT) coefficients, which have been affected by Additive White Gaussian Noise (AWGN).

Since the work of Donoho and Johnstone [1]-[4], there has been a lot of research on the way of defining the threshold levels and their type (i.e. hard or soft threshold). These algorithms usually perform global thresholding of wavelet coefficients by retaining only large coefficients and setting the rest to zero. Thus, they do not present spatial adaptivity and their performance in real life images is not sufficiently effective.

A wide class of image processing algorithms is based on the DWT. The transform coefficients within the subbands can be locally modeled as independent identically distributed (i.i.d) random variables with Generalized Gaussian Distribution (GGD) [5]. In that sense, the denoised coefficients may be evaluated by an MMSE (Minimum Mean Square Error) estimator, in terms of the noised coefficients and the variances of signal and noise. The signal variance is locally estimated by a ML (Maximum Likelihood) estimator, whereas noise variance is estimated from the first level diagonal details. Therefore, the denoised coefficients are statistically estimated in small regions for every subband instead of applying a global threshold [6]. These methods present efficient results but their spatial adaptivity is not well suited near object edges where the variance field is not smoothly varied. In [7] a similar spatially adaptive model for wavelet image coefficients was used to perform image denoising via wavelet thresholding. In [8] the denoised coefficients are statistically estimated in a variable block size framework resulting in a quad-tree decomposition of subbands with respect to local variance.

The present work evaluates the spatially adaptive model as in [6] which performs MMSE coefficient estimation rather than coefficient thresholding as in [7]. The estimation of the underlying variance field is performed in a fixed block size framework employing a ML estimator throughout all the detail subbands of the wavelet decomposition. The fixed blocks are centered square-shaped windows, the size of which affects the estimation of the coefficient variance. The use of different size of window in each decomposition level for improved performance is also considered. The performance of this model with respect to other popular denoising algorithms, as far as reconstruction quality is concerned, is also examined.

This paper is organized as follows. Section 2 is an overview of the denoising algorithms based on the statistical modeling of wavelet coefficients. The experimental results are presented in Section 3 and the conclusions are summarized in Section 4.

2. OVERVIEW

2.1 The statistical model

The statistical model of the proposed denoising algorithm is illustrated in Fig. 1. A noise contaminated image may be formulated as in the shown block-diagram. A "clean" image, x, is decomposed by DWT providing the wavelet coefficients $X(k)$. These coefficients, which may be locally considered as i.i.d GGD random variables with variance $\sigma_X^2(k)$, are corrupted by additive i.i.d Gaussian noise samples, $n(k)$, to produce the observed wavelet coefficients of the noisy image, $Y(k)$.

Let W and W^{-1} denote the two dimensional DWT and its inverse respectively. The relationship between image and transform coefficients is:

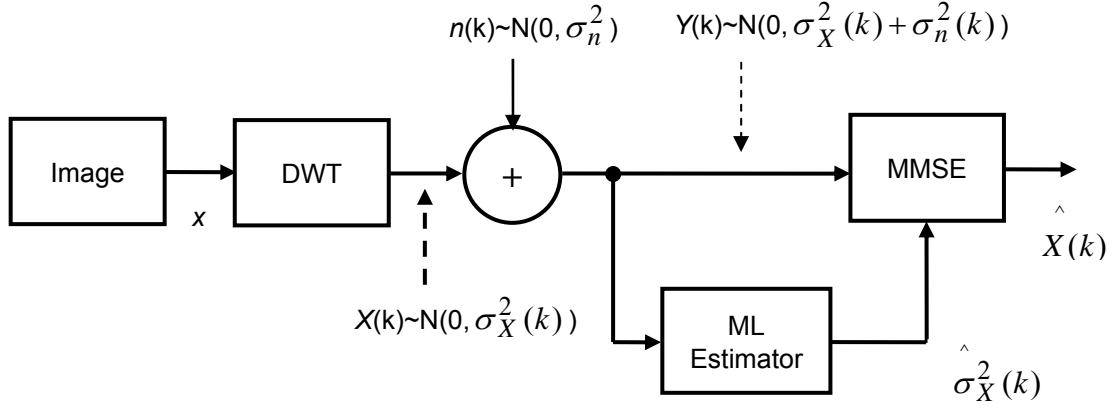$$X = Wx \qquad and \qquad x = W^{-1}X \qquad (1)$$

Fig. 1. Block diagram of the denoising algorithm. The statistical model is based on a ML estimator for an estimate of the underlying variance and a MMSE estimator for the evaluation of the "clean" wavelet coefficients.

The "clean" coefficients, X, may be estimated from the observed coefficients, Y, if noise variance, σ_n^2 and image coefficient variance, $\sigma_X^2(k)$ are known. Here, a robust median estimator of the highest subband diagonal coefficients (i.e. HH$_1$) estimates the noise variance [2]. Also, a ML estimation of image coefficient variance, $\hat{\sigma}_X^2(k)$, is performed using the observed noisy data in small regions that are defined as variable size blocks across scales. Finally, an MMSE estimator provides an estimate of the "clean" coefficients, $\hat{X}(k)$. The reconstructed denoised image is given by:

$$\hat{x} = W^{-1}\hat{X} \tag{2}$$

2.2. The mathematical estimation

It is known that the best estimate of a random variable x given by a MMSE estimator is:

$$\hat{X} = E[X] \tag{3}$$

Also, under the assumptions of independence and Gaussian distribution of the random variables, it is known that [9]:

$$f_X \sim N(0, \sigma_X^2)$$
$$f_Y \sim N(0, \sigma_X^2 + \sigma_n^2) \tag{4}$$
$$f_{Y|X} \sim N(x, \sigma_n^2)$$

The last equation describes the conditional distribution of the observed values when the "clean" values are known. The Bayesian estimation of the "clean" values given the observed values is:

$$f_{X|Y} = \frac{f_{Y|X}\, f_X}{f_Y} = \frac{N(x, \sigma_n^2)\, N(0, \sigma_X^2)}{N(0, \sigma_X^2 + \sigma_n^2)} \tag{5}$$

If the Gaussian distributions are replaced by their explicit forms, equation (5) results in:

$$f_{X|Y} \sim N(\frac{\sigma_X^2}{\sigma_X^2 + \sigma_n^2} y, \frac{\sigma_X^2 \sigma_n^2}{\sigma_X^2 + \sigma_n^2}) \qquad (6)$$

The estimated "clean" wavelet coefficients result by considering equations (3) and (6):

$$X(k) = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_n^2} Y(k) \qquad (7)$$

But in fact σ_X^2 is not known, so we employ a ML estimator in order to have an estimate for a local neighborhood, $\hat{\sigma}_X^2$, where variance is assumed to be constant. The ML estimate is defined as:

$$\hat{X}_{ML}(y) = \arg\max_x f_{Y|X}(y \mid x) \qquad (8)$$

In our case, this takes the following form:

$$\hat{X}_{ML}(y) = \arg\max_{\sigma_y^2} \prod_{j \in N} f(y, \sigma_y^2) \qquad (9)$$

where N is the local neighborhood. The maximum of the above equation is found to be for:

$$\sigma_y^2 = \frac{1}{M} \sum_{k=1}^{M} Y^2(k) \qquad (10)$$

where M represents the number of wavelet coefficients residing in the local neighborhood N.

Therefore, the estimate of the "clean" coefficients variance is:

$$\hat{\sigma}_X^2 = \frac{1}{M} \sum_{k=1}^{M} Y^2(k) - \sigma_n^2 \qquad (11)$$

Finally, the "clean" coefficients are estimated combining (7) and (11):

$$\hat{X}(k) = \frac{\hat{\sigma}_X^2}{\hat{\sigma}_X^2 + \sigma_n^2} Y(k) \qquad (12)$$

where the noise variance is estimated, as it was stated in the previous subsection, by:

$$\sigma_n^2 = \left[\frac{median(|Y(k)|)}{0.6745} \right]^2 \qquad (13)$$

where $Y(k)$ represent the coefficients of HH_1 subband.

3. EXPERIMENTAL RESULTS

The experimental evaluation is performed on three grey scale images like "Lena", "Barbara" and "Boat" of size 512×512 pixels at different noise levels. The wavelet transform employs Daubechies's least asymmetric compactly supported wavelet with eight vanishing moments [10] at five levels of decomposition. The objective quality of the reconstructed image is measured by:

$$PSNR = 10\log_{10}\frac{255^2}{mse} \text{ dB} \qquad (14)$$

where *mse* is the mean square error between the original and the denoised image with size I×J:

$$mse = \frac{1}{IxJ}\sum_{i=1}^{I}\sum_{j=1}^{J}[x(i,j)-\hat{x}(i,j)]^2 \qquad (15)$$

The wavelet domain denoising algorithms compared are SureShrink [3], BayesShrink [7], NormalShrink [11], Wiener [12] and LAWML [6]. The first of the above mentioned methods is the hard-thresholding of wavelet coefficients using a constant threshold for all subbands that is estimated by a robust estimator from HH_1 subband. The second method uses spatially adaptive wavelet thresholding. The third method employs the same principle as the previous one in order to estimate subband dependent threshold. The fourth method is based on the Wiener filter denoising algorithm and the last one employs the statistical modeling of wavelet coefficients in order to estimate the "clean" coefficients using the observed ones and estimating the underlying variance field in a local neighborhood. The PSNR of the various methods are compared in Table I and the best ones are highlighted with bold fonts.

TABLE I
PSNR comparative results for various denoising methods considering various levels of noise strength.

	SureShrink	BayesShrink	NormalShrink	Wiener	LAWML
Lena					
σ_n=10	33.55	33.45	32.86	33.54	**34.54**
σ_n=20	30.41	30.33	29.82	29.00	**31.07**
σ_n=30	28.70	28.65	28.12	25.73	**29.01**
Barbara					
σ_n=10	31.35	31.10	30.46	29.90	**32.87**
σ_n=20	27.43	27.36	26.29	26.79	**28.92**
σ_n=30	25.44	25.41	24.38	24.35	**26.82**
Boat					
σ_n=10	31.86	31.90	30.87	31.71	**32.65**
σ_n=20	28.49	28.43	27.62	28.19	**29.21**
σ_n=30	26.68	26.64	26.01	25.28	**27.20**

It is apparent that LAWML algorithm, which in Table I has been considered with blocks of size 7×7, outperforms all other methods. The size of blocks is unique for all the subbands and its optimum value is image dependent. Table II illustrates the objective quality of LAWML with blocks of size 7×7 compared with variable size

blocks in each decomposition level. In this case, the size of blocks from the finest to the coarsest level is considered to be 11×11, 9×9, 7×7, 5×5, 3×3 respectively.

TABLE II
PSNR comparative results of LAWML denoising algorithm when fixed or variable window size is considered.

	LAWML (fixed size block)	LAWML (variable size block)
Lena		
$\sigma_n=10$	**34.54**	**34.54**
$\sigma_n=20$	31.03	**31.22**
$\sigma_n=30$	29.04	**29.38**
Barbara		
$\sigma_n=10$	**32.85**	32.69
$\sigma_n=20$	**28.97**	28.94
$\sigma_n=30$	26.78	**26.88**
Boat		
$\sigma_n=10$	**32.67**	32.55
$\sigma_n=20$	29.20	**29.22**
$\sigma_n=30$	27.18	**27.32**

The above results show that the efficiency of LAWML method can be improved if a variable block size scheme across the levels of wavelet decomposition is employed. The optimum size of blocks depends on the noise level and the input image. Its adaptivity to these factors can be the objective of a future research work.

(a)   (b)

(c)　　　　　　　　　　　　　　　　　　　(d)

(e)　　　　　　　　　　　　　　　　　　　(f)

Fig. 2. Subjective quality performance comparison for noise standard deviation σ_n=30. (a) Noisy image; (b) SureShrink; (c) BayesShrink; (d) NormalShrink; (e) Wiener filter; (f) LAWML (block size 7×7)

Fig. 2 shows the subjective quality of all the examined methods for "Lena" image, which is affected with noise of σ_n=30. It is obvious that LAWML method performs better denoising than the other methods and provides a more pleasant image quality in a strong additive noise environment.

Fig. 3 shows the subjective quality of the two best methods, which are SureShrink and LAWML with variable block size across the decomposition levels, for "Barbara".The noise standard deviation is set again to σ_n=30. The reconstructed images have been magnified in order to observe differences around a region that contains low and high texture areas. It may be observed that LAWML with variable

block size performs better than SureShrink around the kerchief stripes of "Barbara" as well as around her face, where the reconstructed image has smoother texture.
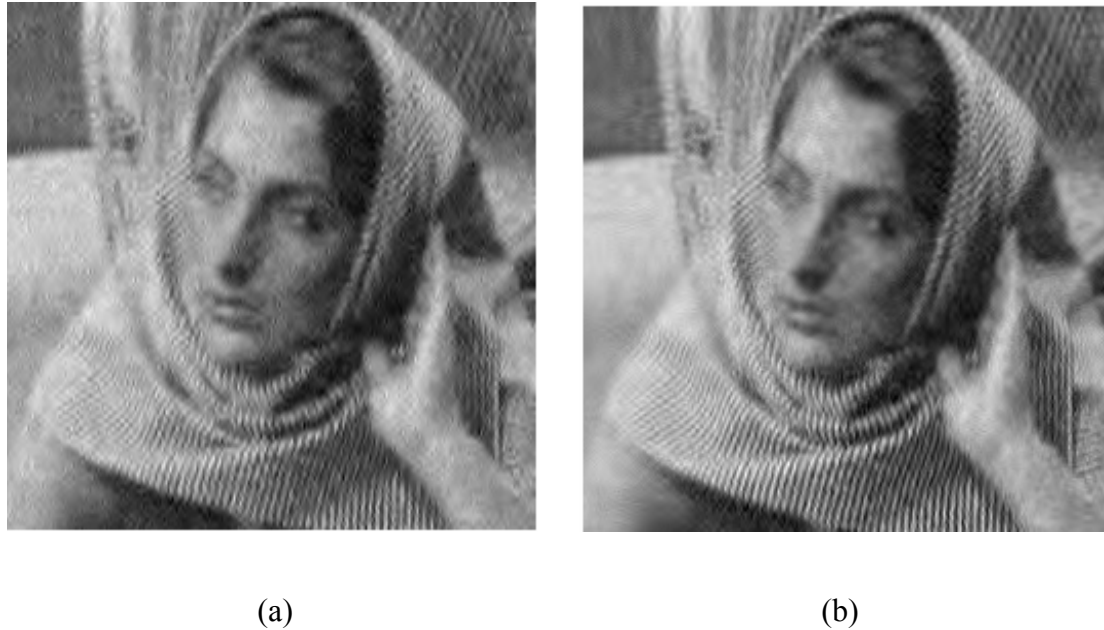

(a)   (b)

Fig. 3. Subjective quality performance comparison for noise standard deviation σ_n=30. (a) SureShrink; (b) LAWML (variable block size)

4. CONCLUSIONS

In this paper, various methods are evaluated for recovering an image from noise contamination effectively. They are based on the discrete wavelet decomposition of the image and the Generalized Gaussian Distribution modeling of the subband coefficients. The most effective method is proved to be LAWML that employs a spatially adaptive model performing MMSE coefficient estimation instead of the classical threshold estimation. The proposed algorithm segments the subbands into blocks of fixed size and estimates the variance in each block assuming that it is smoothly varying in a local neighborhood. The noise variance is estimated by the robust estimator used by SureShrink method. Finally, the "clean" coefficients are estimated by a MMSE estimator and the "clean" image is recovered by an inverse wavelet transform. The use of variable size blocks across the levels of the wavelet decomposition may improve the performance of the specific algorithm, but the optimum choice needs further research.

ACKNOWLEDGMENTS

REFERENCES

[1] D.L. Donoho, Denoising by soft thresholding, IEEE Trans. on Info. Theory, pp. 933-936, 1993.

[2] D.L. Donoho, I.M. Johnstone, Ideal spatial adaptation via wavelet shrinkage, Biometrica, vol. 81, pp. 425-455, 1994.

[3] D.L. Donoho, I.M. Johnstone, Adapting to unknown smoothness via wavelet shrinkage, Journal of American statistical assoc., vol. 90, no. 432, pp. 1200-1224, Dec. 1995.

[4] D.L. Donoho, I.M. Johnstone, Wavelet shrinkage: Asymptopia, J.R. Stat. Soc., series B, vol. 57, no. 2, pp. 301-369, 1995.

[5] S. Mallat, A theory for multiresolution signal decomposition: the wavelet representation, IEEE Trans. on Pattern Anal. Mach. Intell., vol. 11, pp. 674-693, July 1989.

[6] M. K. Mihcak, I. Kozintsev, K. Ramchandran, Spatially adaptive statistical modelling of wavelet image coefficients and its application to denoising, IEEE Intern. Conf. on Acoustics, Speech and Signal Processing, vol.6, pp. 3253-3256, March 1999.

[7] S. G. Chang, B. Yu and M. Vetterli, Adaptive wavelet thresholding for image denoising and compression, IEEE Trans. on Image Proc., vol. 9, no. 9, pp. 1532-1546, Sept. 2000.

[8] J. N. Ellinas, D. E. Manolakis, "Image Denoising based on the Statistical Modeling of Wavelet Coefficinets and Quad-Tree Decomposition", 5th International Conference on Technology and Automation, pp. 341-347, Oct. 2005, Thessaloniki.

[9] R. M. Gray, L. D. Davisson, An introduction to statistical signal processing, Stanford University, 1999.

[10] I. Daubechies, Ten lectures on wavelets, Proceedings CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, vol. 61, 1992.

[11] L. Kaur, S. Gupta, R.C.Chauhan, "Image denoising using wavelet thresholding", Indian Conference on computer Vision, Graphics and Image Processing, Ahmedabad, Dec. 2002.

[12] Mathworks Inc., Matlab, Image Processing Toolbox.